THE DYNAMICS OF VORTEX OSCILLATION IN A SUPERCONDUCTOR

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Abstract

In this paper, we study the phenomenon of magnetic flux oscillations — the oscillation of vortex matter as a result of the thermomagnetic instability of a critical state in a superconductor. The spatial and temporal distributions of thermal and electromagnetic disturbances in a flat semi-infinite superconducting sample in the viscous flow regime with a linear current-voltage characteristic are studied.

Keywords: *vortex matter, oscillation, flow jumps, critical state*

The stability dynamics of the critical state with respect to magnetic flux jumps in hard and composite superconductors was discussed in theoretical and experimental works [1-5]. Total the concept of stability of a critical state in type II superconductors was developed in the literature [4, 5]. In [5], the dynamics of the development of small thermal and electromagnetic disturbances and the corresponding stability conditions for the critical state in superconductors in the viscous flow regime were studied. Recently, great attention has been paid to the phenomenon of magnetic flux oscillations arising as a result of thermomagnetic instability in superconductors [6]. In the process of studying the dynamics of thermomagnetic instabilities, vibrational modes in the mixed state of a superconducting Nb-Ti sample were detected as a result of a catastrophic avalanche [7]. To explain the observed oscillation processes, a theoretical model was proposed that takes into account the inertial properties of vortex matter [8]. In [9], the dynamic properties of vortex matter in an Nb-Ti superconductor were studied. Oscillation phenomena were interpreted as the result of the existence of a finite value of the effective mass of the vortex, i.e. oscillations can be considered as a manifestation of the inertial properties of vortex matter [10]. In this work, the phenomenon of magnetic flux oscillations as a result of thermomagnetic instability of a critical state in a superconductor is theoretically investigated.

The system of equations of macroscopic electrodynamics is used to simulate the evolution of temperature and electromagnetic field perturbations. The distribution of magnetic induction $\vec{B}(r, t)$ and transport current $\vec{j}(r, t)$ in a superconductor is given by the equation

$$\text{rot}\,\vec{B}=\frac{4\pi}{c}\vec{j} \tag{1}$$

In viscous flow regime, the relationship between magnetic induction $\vec{B}(r,\ t)$ and electric field $\vec{E}(r,\ t)$ is established by Maxwell's equations

$$\text{rot}\,\vec{E}=-\frac{1}{c}\frac{d\vec{B}}{dt} \tag{2}$$

$$\vec{E}=\frac{v}{c}\vec{B} \tag{3}$$

The equation of motion of the vortices can be written in the form [6].

$$m\frac{dV}{dt}+\pi V+F_L+F_P=0 \tag{4}$$

where m is the mass of the vortex of unit length, $F_L=\frac{1}{c}\vec{j}\,\vec{\Phi}_0$ is the Lorentz force, $\eta=\frac{\vec{\Phi}_0 H_{C2}}{c^2\rho_n}$ is the viscosity coefficient, ρ_n is the resistance in the normal state, $\vec{\Phi}_0=\frac{\pi hc}{2e}$ is the magnetic flux quantum, is the upper critical field [1]. Accordingly, the temperature distribution in the sample is determined by the heat equation

$$\nu\frac{dT}{dt}=\Delta\left[\kappa(T)\Delta T\right]+\vec{j}\vec{E} \tag{5}$$

where $\nu=\nu(T)$ and $\kappa=\kappa(T)$ are the heat capacity and thermal conductivity coefficients of the sample, respectively. We use the Bean model for the current density $\vec{j}(T,E,B)$ and assume that it does not depend on the magnetic field induction, $j=j_C(B_e,T)$, i.e., $j_C=j_0-a\left(T-T_0\right)$ [1], ie, [1], where B_e is the value of the external magnetic induction; $a=j_0/\left(T_C-T_0\right)$; j_0 is the equilibrium current density, T_0 and T_C are the initial and critical temperature of the sample, respectively [5]. We assume that the external magnetic field $\vec{B}=(0,\ 0,\ B_e)$ is directed along the z axis and the magnetic field velocity is constant $\dot{B}_e=\text{const}$.. For small thermal and electromagnetic disturbances $\Theta(x,t),\ b(x,t),\ v(x,t)\cdot\exp\left(\gamma t\right)$ (where γ is the eigenvalue of the problem), it is easy to obtain the dispersion relation that determines the eigenvalue of the problem

$$\frac{d^2b}{dx^2}-\left[(\gamma+\beta)\mu-2\beta\right]\frac{db}{dx}+\left[(\mu+1)\gamma^2+(\mu-1)\beta-(\mu-1)\beta\right]b=0 \quad (6)$$

where dimensionless parameters were introduced $\mu=\frac{c\vec{\Phi}_0}{4\pi\eta^2}\frac{B_e}{2L^2}$, $b=\frac{B}{B_e}=\frac{c}{4\pi}\frac{B}{j_C L}$, $\Theta=\frac{4\pi}{c}\frac{2\nu}{B_e^2}$, $v=V\frac{t_0}{L}$, $L=\frac{c}{4\pi}\frac{B_e}{j_C}$ and $z=\frac{x}{L}$, $\mu=\frac{c\vec{\Phi}_0}{4\pi\eta^2}\frac{B_e}{2L^2}$ $\tau=\frac{t}{t_0}=\frac{c\vec{\Phi}_0}{4\pi\eta}\frac{B_e}{2\mu_0 j_C L^2}$. Variables.

Here L is the depth of penetration of the magnetic field deep into the superconductor [5]. The instability of the magnetic front, as a rule [5], is determined by the positive values of the increment $\mathrm{Re}\,\gamma\geq 0$. Then we can assume that the instability arises under the condition $\mathrm{Re}\,\gamma=0$. Analysis of the dispersion relation shows that the growth increment is positive $\mathrm{Re}\,\gamma\geq 0$ if the condition $\mu\geq\mu_C=2$ is met. In this case $\mu\geq\mu_C$, small perturbations increase with time and the magnetic flux front is unstable. In the case when the growth increment is negative $\mu\geq\mu_C$ and any small perturbation will decay. At a critical value, the growth increment is zero $\gamma=0$ [11].

In the particular case when μ = 1, the rise parameter is determined by the stability parameter β> 0. Then, the stability criterion can be represented as β> 1. In another particular case, when the thermal effects are insignificant (β = 1), the following dispersion relation can be obtained

$$\frac{d^2b}{dx^2}-\mu\frac{db}{dx}+\left[(\gamma-1)(\mu+1)\right]b=0$$

Representing the solution of the dispersion equation in the form

$$b \sim \exp(ikx)$$

we can obtain the dependence of the growth parameter γ on the wave vector k. An analysis shows [1] that when k <kc = μ, the growth increment is positive and a small perturbation increases with time. For the values of the wave vector k> k_c, the quantity γ is negative and the small perturbation decays exponentially. It can be shown [11] that for k = k_c the growth increment is γ = 0. If the wave vector tends to zero k → 0 or infinity k → ∞, the quantity γ = 1 and a small perturbation increases. In this case, the quantity γ is determined by the value

$$\gamma = \frac{2\mu}{\mu + 1}$$

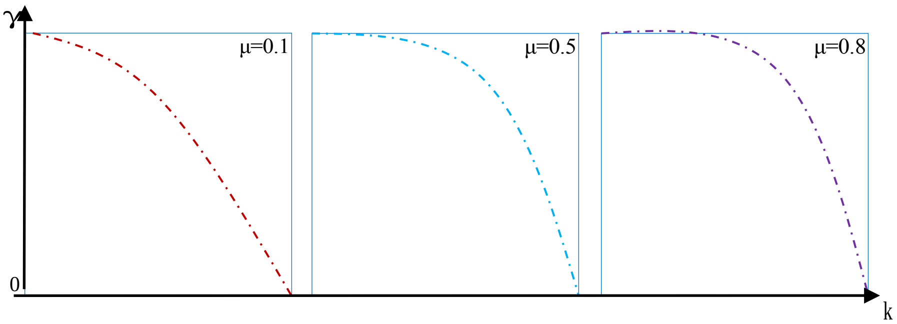

Рисунок 1 *Зависимость параметра нарастания от волнового вектора для различных значений μ=0.1, 0.5, 0.8*

For $\mu = 0$, the value of the growth increment is $\gamma = 0$. For $\mu = 1$, the value of $\gamma = 1$. The dependence of the growth rate of γ on the wave vector is shown in Fig. 1. for various values of the parameter μ. As μ increases, the parameter γ increases. At certain values of the parameter μ, jumps in the flow are observed, which take into account the inertial properties of the vortices.

CONCLUSION

Thus, in this work, the phenomenon of magnetic flux oscillations is studied theoretically — the oscillation of vortex matter as a result of the thermomagnetic instability of a critical state in a superconductor. The spatial and temporal distributions of thermal and electromagnetic disturbances in a flat semi-infinite superconducting sample in the viscous flow regime with a linear current-voltage characteristic are studied.

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